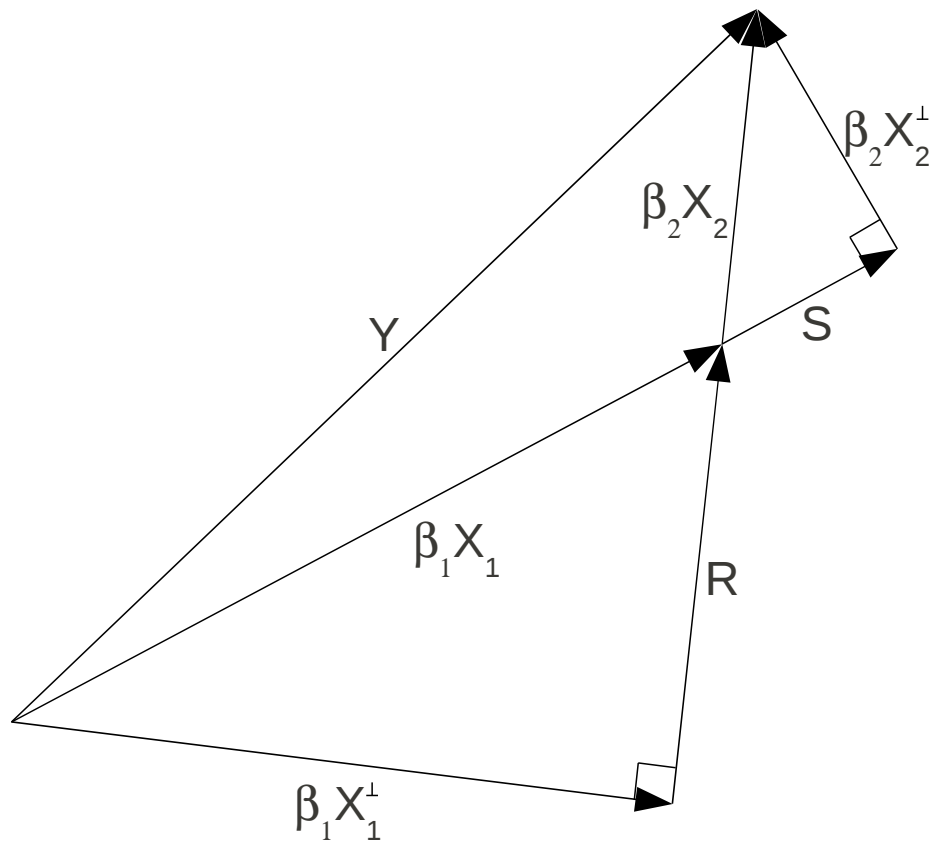


# Correlated Regressors in SPM: a (hopefully not overly) simplified view



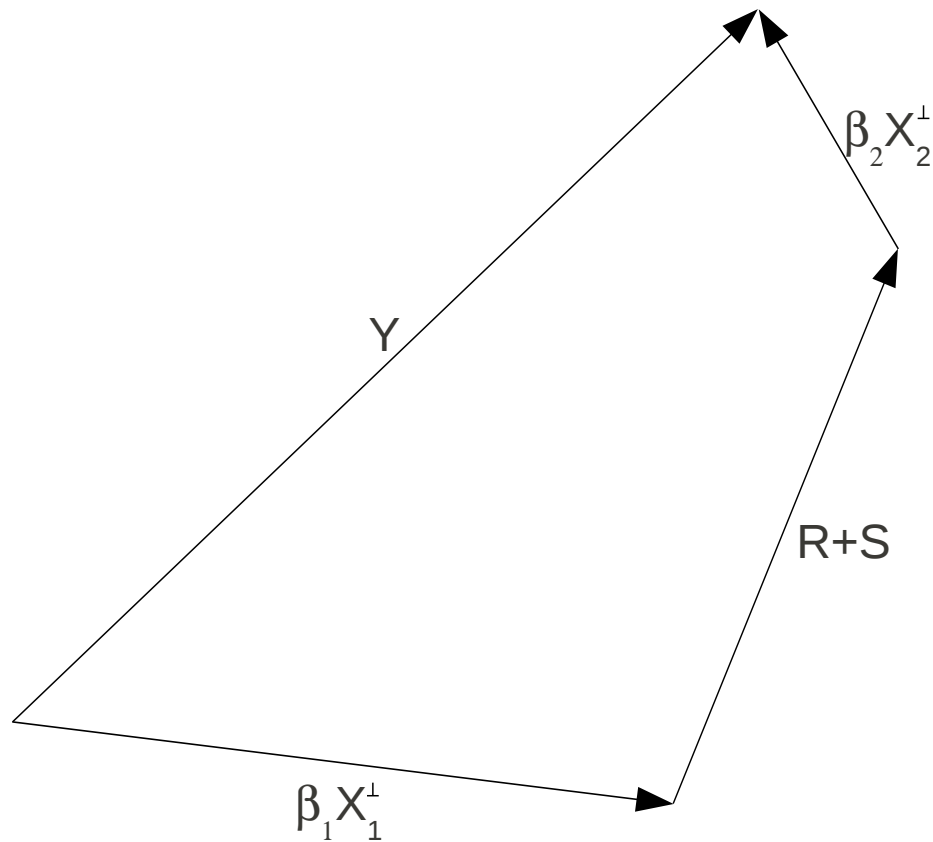
- $Y$  is the time series projected onto the space spanned by  $X_1$  and  $X_2$ .
- $X_1$  and  $X_2$  are two correlated regressors.
- $X_1^\perp$  is the result of orthogonalizing  $X_1$  with respect to  $X_2$ .
- $R$  and  $S$  represent the shared variance; they can be explained by either  $X_1$  or  $X_2$ . For example, for some scalar  $r$ ,

$$R = X_1 - X_1^\perp = rX_2$$

- Note that orthogonalizing one of the regressors does not affect the  $\beta$  value estimated by SPM for that regressor. Instead, the  $\beta$  for the other regressor changes. That is to say, if  $X_2$  is orthogonalized,

$$\begin{aligned} Y &= \beta_1 X_1 + S + \beta_2 X_2^\perp \\ &= (\beta_1 + s)X_1 + \beta_2 X_2^\perp \end{aligned}$$

- So the new  $\beta$  for  $X_1$  is  $(\beta_1 + s)$ , reflecting the fact that the shared variance has been assigned to  $X_1$ .  $s$  is a scalar such that  $S = sX_1$ .



- When SPM estimates the model, it puts the shared variance  $R + S$  into the error term, instead of arbitrarily assigning some to  $X_1$  and some to  $X_2$ .
- So the  $\beta$ s only tell you what is modeled solely by the corresponding regressor (i.e. what can't be modeled by any of the other regressors). This is important when interpreting the results of a model containing correlated regressors.
- SPM automatically orthogonalizes parametric modulators, but main conditions are unchanged and therefore subject to the above considerations.
- The orthogonalization of the PMs is done left to right as determined by the order in which they were entered into the model: the second PM is orthogonalized with respect to the first, then the third PM is orthogonalized with respect to the first two, and so on. Hence any shared variance is assigned to the leftmost appropriate PM.